

MATH 1065 - Chapter 6 Review

6.1 (66) $-4 \tan^{-1} x = \pi$

$\tan^{-1} x = -\pi/4$
 $\tan(-\pi/4) = x$
 $-1 = x$

6.3 (15) $[0, 2\pi)$
 $2 \sin^2 \theta - 1 = 0$

$2 \sin^2 \theta = 1$
 $\sin^2 \theta = 1/2$
 $\sin \theta = \pm \sqrt{1/2} = \pm \frac{\sqrt{2}}{2}$
 $\theta = \pi/4$ (I, II, III, IV)
 $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

(43) $\sin(\theta/2) = -\frac{\sqrt{3}}{2}$

$(\theta/2) = \pi/3$ (III, IV)
 $\theta/2 = \frac{4\pi}{3} + 2k\pi$
 $\theta = \frac{8\pi}{3} + 4k\pi$ $\frac{10\pi}{3} + 4k\pi$

(60) $[0, 2\pi)$
 $2 \cos^2 \theta + \cos \theta - 1 = 0$

$(2 \cos \theta - 1)(\cos \theta + 1) = 0$
 $\cos \theta = 1/2$ $\cos \theta = -1$
 $\theta = \pi/3$ (I, IV)
 $\theta = \pi/3, 5\pi/3, \pi$

(78) $\csc^2 \theta = \cot \theta + 1$ $[0, 2\pi)$

$\cot^2 \theta + 1 = \cot \theta + 1$
 $\cot^2 \theta - \cot \theta = 0$
 $\cot \theta (\cot \theta - 1) = 0$
 $\cot \theta = 0$ $\cot \theta = 1$
 $\theta = \pi/2, 3\pi/2$ (I, III)
 $\theta = \pi/4, 5\pi/4$

6.4 (56) $\frac{\cot \theta}{1 - \tan \theta} + \frac{\tan \theta}{1 - \cot \theta} = 1 + \tan \theta + \cot \theta$

$\frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} + \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}}$
 $\frac{\cos^2 \theta}{\sin \theta \cos \theta - \sin^2 \theta} + \frac{\sin^2 \theta}{\sin \theta \cos \theta - \cos^2 \theta}$
 $\frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)} + \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)}$
 $\frac{-\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)}$
 $\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)}$
 $\frac{1 + \sin \theta \cos \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} + 1$

$1 + \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$
 $1 + \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$
 $1 + \frac{1}{\sin \theta \cos \theta}$

(81) $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta$

$\frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)}{(\sin \theta + \cos \theta)}$
 $1 - \sin \theta \cos \theta$

6.6 (24) Use $\frac{1}{2}$ -Angle Identity

$\sin 195^\circ = \sin \left[\frac{1}{2}(390^\circ) \right]$
 $= + \sqrt{\frac{1 - \cos 390^\circ}{2}}$

$= - \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = - \sqrt{\frac{2 - \sqrt{3}}{4}} = \left(- \frac{\sqrt{2 - \sqrt{3}}}{2} \right)$

(54) $(4 \sin u \cos u)(1 - 2 \sin^2 u) = \sin(4u)$
 $[2(2 \sin u \cos u)](1 - 2 \sin^2 u)$
 $[2 \sin(2u)] \cos(2u)$
 $\sin(4u)$

6.7) ⑥ $\sin(4\theta) \cos(6\theta) = \frac{1}{2} [\sin(10\theta) + \sin(-2\theta)] = \frac{1}{2} \sin(10\theta) - \frac{1}{2} \sin(2\theta)$

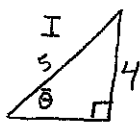
⑬ $\cos \theta + \cos(3\theta) = 2 \cos\left(\frac{4\theta}{2}\right) \cos\left(\frac{-2\theta}{2}\right) = 2 \cos(2\theta) \cos(-\theta) = 2 \cos(2\theta) \cos \theta$


⑭ $\sin\left(\frac{\theta}{2}\right) - \sin\left(\frac{3\theta}{2}\right) = 2 \cos(\theta) \sin\left(-\frac{\theta}{2}\right) = -2 \cos(\theta) \sin\left(\frac{\theta}{2}\right)$

6.R ④ $\sin^{-1}\left(-\frac{1}{2}\right) \bar{\theta} = \frac{\pi}{6} \theta = -\frac{\pi}{6}$

⑦ $\sec^{-1} \sqrt{2} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

⑪ $\tan^{-1}\left(\tan \frac{2\pi}{3}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

⑲ $\sin\left(\cot^{-1} \frac{3}{4}\right) = \sin(\theta)$  $\sin(\theta) = \frac{4}{5}$

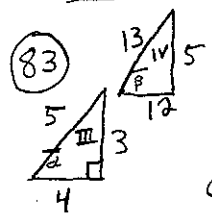
⑳ $\sin\left(\csc^{-1} u\right) = \sin(\theta)$  $\sin \theta = \frac{1}{u}$

④⑥ $4 \sin^2 \theta + 2 \cos^2 \theta = 4 - 2 \cos^2 \theta$
 $2 \sin^2 \theta + 2 \sin^2 \theta + 2 \cos^2 \theta$
 $+ 2 [\sin^2 \theta + \cos^2 \theta]$
 $2 \sin^2 \theta + 2$
 $2 [1 - \cos^2 \theta] + 2$
 $2 - 2 \cos^2 \theta + 2$
 $4 - 2 \cos^2 \theta \checkmark$

⑥③ $(1 + \cos \theta) \tan \frac{\theta}{2} = \sin \theta$
 $(1 + \cos \theta) \left[\frac{\sin \theta}{1 + \cos \theta} \right] = \sin \theta \checkmark$

⑦③ $\sin 165^\circ = \sin(120^\circ + 45^\circ)$
 $= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$
 $= \frac{\sqrt{3}}{2} \left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right)$
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$

⑦⑦ $\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$



a) $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \left(-\frac{3}{5}\right) \left(\frac{12}{13}\right) + \left(-\frac{4}{5}\right) \left(-\frac{5}{13}\right) = \frac{-16}{65}$
 b) $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \left(-\frac{4}{5}\right) \left(\frac{12}{13}\right) - \left(-\frac{3}{5}\right) \left(-\frac{5}{13}\right) = -\frac{63}{65}$
 c) $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta = \left(-\frac{3}{5}\right) \left(\frac{12}{13}\right) - \left(-\frac{4}{5}\right) \left(-\frac{5}{13}\right) = \frac{-56}{65}$

d) $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{3}{4} - \frac{5}{12}}{1 - \frac{3}{4} \left(-\frac{5}{12}\right)} = \frac{16}{63}$

e) $\sin(2\alpha) = 2 \sin \alpha \cos \alpha = 2 \left(-\frac{3}{5}\right) \left(-\frac{4}{5}\right) = \frac{24}{25}$

f) $\cos(2\beta) = 2 \cos^2 \beta - 1 = 2 \left(\frac{12}{13}\right)^2 - \frac{169}{169} = \frac{119}{169}$

$180^\circ < \alpha < 270^\circ$ $270^\circ < \beta < 360^\circ$
 $90^\circ < \frac{\alpha}{2} < 135^\circ$ $135^\circ < \frac{\beta}{2} < 180^\circ$

g) $\sin\left(\frac{\beta}{2}\right) = + \sqrt{\frac{1 - \cos \beta}{2}} = + \sqrt{\frac{1 - \frac{12}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{\sqrt{26}}{26}$

h) $\cos\left(\frac{\alpha}{2}\right) = - \sqrt{\frac{1 + \cos \alpha}{2}} = - \sqrt{\frac{1 - \frac{4}{5}}{2}} = - \sqrt{\frac{1}{10}} = -\frac{1}{\sqrt{10}}$

⑨④ $\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right] = \cos[A + B] = \cos A \cos B - \sin A \sin B = \left(\frac{1}{\sqrt{2}}\right) \left(-\frac{4}{5}\right) - \left(-\frac{1}{\sqrt{2}}\right) \left(\frac{3}{5}\right) = \frac{-4 + 3}{5\sqrt{2}} = -\frac{\sqrt{2}}{10}$

⑨⑥ $\cos\left(2 \tan^{-1} \frac{4}{3}\right) = \cos(2\theta) = 2 \cos^2 \theta - 1 = 2 \left(\frac{3}{5}\right)^2 - 1 = \frac{-7}{25}$

⑪② $[0, 2\pi) \sin(2\theta) - \sin \theta - 2 \cos \theta + 1 = 0$

$2 \sin \theta \cos \theta - \sin \theta - 2 \cos \theta + 1 = 0$
 $\sin \theta (2 \cos \theta - 1) - 1 (2 \cos \theta - 1) = 0 \Rightarrow (\sin \theta - 1)(2 \cos \theta - 1) = 0$
 $\sin \theta = 1 \quad \cos \theta = \frac{1}{2}$
 $\bar{\theta} = \frac{\pi}{3}$
 $\theta = \frac{\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

⑪⑤ $\sec^{-1}(3) = \cos^{-1}\left(\frac{1}{3}\right) = 1.23096 \text{ Radians or } 70.529^\circ$