MATH 1220 - Review for Exam 10

10.4 (14) $\vec{a} = \langle 2, 6t, 12t^2 \rangle$ $v(0) = \langle 1, 0, 0 \rangle$ $r(0) = \langle 0, 1, -1 \rangle$

$v(t) = \langle 2t+1, 3t^2, 4t^3 \rangle \Rightarrow r(t) = \langle t^2+t, t^3+1, t^4-1 \rangle$

(17) $r(t) = \langle t^2, 5t, t^2-16t \rangle$ Find when SPEED is minimized?

$\vec{v}(t) = \langle 2t, 5, 2t-16 \rangle$ $v = \text{Speed} = \sqrt{4t^2+25+(4t^2-64t+256)}$

$v = \sqrt{8t^2-64t+281}$ $v' = \dfrac{8t-32}{\sqrt{8t^2-64t+281}}$ $\underset{}{-}\ \underset{4}{+}$ v'

Speed is Minimized at $t = 4$

(33) $r(t) = \langle t^3, t^2 \rangle$ $r' = \langle 3t^2, 2t \rangle$ $r'' = \langle 6t, 2 \rangle$

$r' \cdot r'' = 18t^3 + 4t$

$r' \times r'' = \langle 0, 0, 6t^2 - 12t^2 \rangle$ ($-6t^2$)

$a_T = \dfrac{18t^3+4t}{\sqrt{9t^4+4t^2}}$ OR $\dfrac{18t^2+4}{\sqrt{9t^2+4}}$

$a_N = \dfrac{6t^2}{\sqrt{9t^4+4t^2}}$ OR $\dfrac{6t}{\sqrt{9t^2+4}}$

10.R (2) $\vec{r}(t) = \left\langle \sqrt{2-t}, \dfrac{e^t-1}{t}, \ln(t+1) \right\rangle$

a) DOMAIN $\begin{cases} x(t): & t \le 2 \\ y(t): & t \neq 0 \\ z(t): & t > -1 \end{cases}$ $(-1, 0) \cup (0, 2]$

b) $\lim_{t \to 0} \vec{r}(t) = \langle \sqrt{2}, 1, 0 \rangle$

$\lim_{t \to 0} \dfrac{e^t-1}{t} = \dfrac{0}{0} \overset{H}{\Rightarrow} \lim_{t \to 0} \dfrac{e^t}{1} = 1$

c) $r'(t) = \left\langle \dfrac{-1}{2\sqrt{2-t}}, \dfrac{te^t-(e^t-1)}{t^2}, \dfrac{1}{t+1} \right\rangle$

(5) $r(t) = \langle t^2, t\cos \pi t, \sin \pi t \rangle$

$\int_0^1 r(t)\,dt = \left\langle \frac{1}{3}t^3, \dfrac{\cos \pi t + \pi t \sin \pi t}{\pi^2}, -\frac{1}{\pi}\cos \pi t \right\rangle \Big]_0^1 = \left\langle \frac{1}{3}, -\frac{2}{\pi^2}, \frac{2}{\pi} \right\rangle$

(8) $r(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle \quad [0,1]$

$r' = \langle 3t^{1/2}, -2\sin 2t, 2\cos 2t \rangle$

$|r'| = \sqrt{9t+4}$

$L = \int_0^1 \sqrt{9t+4}\, dt = \frac{26\sqrt{13} - 16}{27}$

(11) $r(t) = \langle \frac{1}{3}t^3, \frac{1}{2}t^2, t \rangle \quad r' = \langle t^2, t, 1 \rangle \quad |r'| = \sqrt{t^4+t^2+1}$

$T(t) = \frac{1}{\sqrt{t^4+t^2+1}} \langle t^2, t, 1 \rangle \quad T' = \left\langle \frac{t(t^2+2)}{(t^4+t^2+1)^{3/2}}, \frac{1-t^4}{(t^4+t^2+1)^{3/2}}, \frac{-2t^3-t}{(t^4+t^2+1)^{3/2}} \right\rangle$

$N(t) = \frac{T'}{|T'|} =$

$r'' = \langle 2t, 1, 0 \rangle \quad r' \times r'' = \langle -1, 2t, -t^2 \rangle$

$|r' \times r''| = \sqrt{t^4+4t^2+1} \quad |r'| = \sqrt{t^4+t^2+1}$

$K(t) = \frac{\sqrt{t^4+4t^2+1}}{(t^4+t^2+1)^{3/2}}$

(13) $y = x^4$ at $(1,1)$ $\quad K(x) = \frac{12x^2}{\{1+(4x^3)^2\}^{3/2}} = \frac{12x^2}{(1+16x^6)^{3/2}}$

$F' = 4x^3$

$F'' = 12x^2$

(17) $r(t) = \langle t\ln t, t, e^{-t} \rangle \quad \vec{v} = \langle \ln t + 1, 1, -e^{-t} \rangle \quad v = \sqrt{\ln^2 t + 2\ln t + e^{-2t} + 2}$

$\vec{a} = \langle \frac{1}{t}, 0, e^{-t} \rangle$

$r(0) = \langle 0,0,0 \rangle$

(18) $v(0) = \langle 1, -1, 3 \rangle \quad a(t) = \langle 6t, 12t^2, -6t \rangle$

$\vec{v}(t) = \langle 3t^2+1, 4t^3-1, -3t^2+3 \rangle$

$\vec{r}(t) = \langle t^3+t, t^4-t, -t^3+3t \rangle$