MATH 1220 - Chapter 9 Review

(1) $P(6,-2,3)$ $C(-1,2,1)$ a) SPHERE: $r = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} = \sqrt{7^2+4^2+2^2} = \sqrt{69}$

$\boxed{(x+1)^2 + (y-2)^2 + (z-1)^2 = 69}$

b) yz-plane $(x=0)$

$(0+1)^2 + (y-2)^2 + (z-1)^2 = 69$

$\boxed{(y-2)^2 + (z-1)^2 = 68}$

c) $x^2 - 8x + y^2 + 2y + z^2 + 6z = -1$

$(x-4)^2 + (y+1)^2 + (z+3)^2 = -1 + 16 + 1 + 9$

CENTER: $(4,-1,-3)$ RADIUS: $r^2 = 25 \rightarrow \boxed{r=5}$

(3) a) $\vec{u}\cdot\vec{v} = |u||v|\cos\theta$
$= 2(3)\cos 45^\circ$
$= \boxed{3\sqrt{2}}$

b) $|\vec{u}\times\vec{v}| = |u||v|\sin\theta$
$= 2(3)\sin 45^\circ$
$= \boxed{3\sqrt{2}}$

c) $\vec{u}\times\vec{v}$ is directed OUT of page (Right-Hand Rule)

(4) $\vec{a} = \langle 1,1,-2\rangle$ $\vec{b} = \langle 3,-2,1\rangle$ $\vec{c} = \langle 0,1,-5\rangle$

a) $2\vec{a} + 3\vec{b} = \langle 2,2,-4\rangle + \langle 9,-6,3\rangle = \langle 11,-4,-1\rangle$

b) $|\vec{b}| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$

c) $\vec{a}\cdot\vec{b} = 1(3) + 1(-2) + (-2)1 = -1$

d) $\vec{a}\times\vec{b} = \langle 1-4, -6-1, -2-3\rangle = \langle -3,-7,-5\rangle$

e) $|\vec{b}\times\vec{c}| = |\langle 10-1, 0+15, 3+0\rangle| = \sqrt{81+225+9}$
$= 3\sqrt{35}$

f) $\vec{a}\cdot(\vec{b}\times\vec{c}) = \langle 1,1,-2\rangle\cdot\langle 9,15,3\rangle = 9+15-6 = 18$

g) $C\times C = \langle 0,1,-5\rangle\langle 0,1,-5\rangle = \langle -5+5, 0+0, 0+0\rangle = \vec{0}$

Can also obtain with MAGNITUDE Formula

h) $\vec{a}\times(\vec{b}\times\vec{c}) = \langle 1,1,-2\rangle\times\langle 9,15,3\rangle = \langle 3+30, -18-3, 15-9\rangle = \langle 33,-21,6\rangle$

i) $\text{comp}_a\vec{b} = \frac{a\cdot b}{|a|} = \frac{-1}{\sqrt{1+1+4}} = -\frac{1}{\sqrt{6}}$

j) $\text{proj}_a\vec{b} = \frac{a\cdot b}{|a|^2}\vec{a} = \frac{\text{comp}_a\vec{b}}{|a|}\vec{a} = -\frac{1}{6}\vec{a} = \langle -\frac{1}{6}, -\frac{1}{6}, \frac{1}{3}\rangle$

k) $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|} = \frac{-1}{\sqrt{6}\sqrt{14}} \Rightarrow \theta \approx 96.264$ or 96°

(5) $\vec{a} = \langle 3,2,x\rangle$ $\vec{b} = \langle 2x,4,x\rangle$

Use $\cos\theta = \frac{a\cdot b}{|a||b|}$ ($\cos\theta = \cos(\frac{\pi}{2}) = 0$ If $\vec{a}$ & $\vec{b}$ are orthogonal)

$\Rightarrow \vec{a}\cdot\vec{b} = 0 = 6x + 8 + x^2$

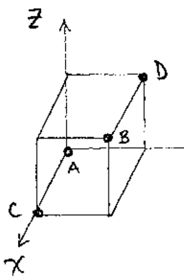
$0 = (x+4)(x+2)$

$\boxed{x=-4 \text{ or } x=-2}$

(6) $\vec{a} = \langle 0,1,2\rangle$ $\vec{b} = \langle 1,-2,3\rangle$

$\vec{a}\times\vec{b} = \langle 7,2,-1\rangle \Rightarrow$ Normalize $\Rightarrow \pm\frac{1}{3\sqrt{6}}\langle 7,2,-1\rangle$

(9)



A(0,0,0)
B(1,1,1)
C(1,0,0)
D(0,1,1)

$\vec{AB}\cdot\vec{CD} = \langle 1,1,1\rangle\cdot\langle -1,1,1\rangle = 1$

$\cos\theta = \frac{\vec{AB}\cdot\vec{CD}}{|AB||CD|} = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$

$\theta = \cos^{-1}(\frac{1}{3}) \approx 70.529^\circ$

(10) $A(1,0,1)$ $B(2,3,0)$ $C(-1,1,4)$ $D(0,3,2)$

$\vec{AB} = \langle 1,3,-1\rangle$
$\vec{AC} = \langle -2,1,3\rangle$
$\vec{AD} = \langle -1,3,1\rangle$

$V(\text{Parallelepiped}) = |\vec{AB}\cdot(\vec{AC}\times\vec{AD})|$

$= |\langle 1,3,-1\rangle\cdot\langle -8,-1,-5\rangle| = |-8-3+5| = 6$

(11) $A(1,0,0)$ $B(2,0,-1)$ $C(1,4,3)$

$\vec{AB} = \langle 1,0,-1\rangle$ $\vec{AC} = \langle 0,4,3\rangle$

$\vec{AB}\times\vec{AC} = \vec{n} = \langle 4,-3,4\rangle$ (a)

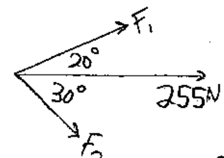
b) $A_\Delta = \frac{1}{2}|\vec{n}| = \frac{1}{2}\sqrt{16+9+16} = \frac{\sqrt{41}}{2}$

(12) $\vec{F} = \langle 3,5,10\rangle$ From $(1,0,2)$ to $(5,3,8)$

$\vec{D} = \langle 4,3,6\rangle$

$W = \vec{F}\cdot\vec{D} = 12+15+60 = 87$ Joules

(13)



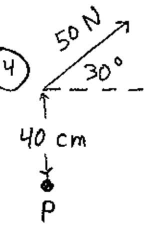
$\vec{F}_1 + \vec{F}_2 = \langle 255, 0\rangle$

$\langle F_1\cos 20^\circ, F_1\sin 20^\circ\rangle + \langle F_2\cos(-30^\circ), F_2\sin(-30^\circ)\rangle = F$

$\cos 20^\circ F_1 + \cos(-30^\circ)F_2 = 255$
$\sin 20^\circ F_1 + \sin(-30^\circ)F_2 = 0$

$F_1 \approx 166.44$ N
$F_2 \approx 113.85$ N

(14)



$|\tau| = |\vec{r}\times\vec{F}| = |\vec{r}||\vec{F}|\sin\theta$

$= .4(50)\sin 60^\circ = 10\sqrt{3}$ Joules

≈ 17.3205

(15) $P(1,2,4)$ $v = \langle 2,-1,3\rangle$

$x = 1+2t$ $y = 2-t$ $z = 4+3t$

(16) $P_1(-6,-1,0)$ $P_2(2,-3,5)$

$v = \langle 8,-2,5\rangle$

$x = -6+8t$ $y = -1-2t$ $z = 5t$

(17) $P(1,0,1)$ parallel to $\begin{cases} x=4t \\ y=1-3t \\ z=2+5t \end{cases}$

$\vec{v} = \langle 4,-3,5 \rangle$

$x = 1+4t \quad y = -3t \quad z = 1+5t$

(18) $P(4,-1,-1) \quad n = \langle 2,6,-3 \rangle$

$2(x-4) + 6(y+1) - 3(z+1) = 0 \Rightarrow 2x+6y-3z=5$

(19) $P(-4,1,2)$ parallel to $x+2y+5z=3$

$n = \langle 1,2,5 \rangle \quad 1(x+4) + 2(y-1) + 5(z-2) = 0$

$x+2y+5z=8$

(20) $A(-1,2,0) \quad B(2,0,1) \quad C(-5,3,1)$

$\vec{AB} = \langle 3,-2,1 \rangle \quad \vec{AC} = \langle -4,1,1 \rangle$

$\vec{n} = \vec{AB} \times \vec{AC} = \langle -3,-7,-5 \rangle \Rightarrow$ use $\langle 3,7,5 \rangle$

$3(x+1) + 7(y-2) + 5(z-0) = 0 \Rightarrow 3x+7y+5z=11$

(21) Contains intersection of $\begin{cases} x-z=1 & (A) \\ y+2z=3 & (B) \end{cases}$ and $\perp$ to: $x+y-2z=1$ (C)

Let $\vec{u} = n_A \times n_B = \langle 1,0,-1 \rangle \times \langle 0,1,2 \rangle = \langle 1,-2,1 \rangle$

Since $\vec{u}$ and n_C are both PARALLEL to desired plane,

$\vec{n} = \vec{u} \times \vec{n_c} = \langle 1,-2,1 \rangle \times \langle 1,1,-2 \rangle = \langle 3,3,3 \rangle$
$\Rightarrow$ Use $\langle 1,1,1 \rangle$

To find P, Let $x=0$ in (A) + (B) $\Rightarrow P = (0,5,-1)$

$1(x-0) + 1(y-5) + 1(z+1) = 0 \Rightarrow x+y+z=4$

(22) $\ell: x = 2-t,\ y = 1+3t,\ z = 4t$; Intersection with $2x-y+z=2$

$2(2-t) - (1+3t) + (4t) = 2 \Rightarrow t = 1 \quad P = (1,4,4)$

(23) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ AND $\frac{x+1}{6} = \frac{y-3}{-1} = \frac{z+5}{2}$

$x = 1+2t \quad y = 2+3t \quad z = 3+4t$ AND $x = -1+6s \quad y = 3-s \quad z = -5+2s$

NOT PARALLEL $\quad \left.\begin{array}{l} x_1 = x_2 \Rightarrow 1+2t = -1+6s \\ y_1 = y_2 \Rightarrow 2+3t = 3-s \end{array}\right\} \begin{array}{l} t = 1/5 \\ s = 2/5 \end{array}$

For such t and s, $z_1 = \frac{19}{5}$ and $z_2 = -\frac{21}{5} \Rightarrow$ SKEW LINES

(24) $x+y-z=1$ AND $2x-3y+4z=5$

b) $\vec{n}_1 = \langle 1,1,-1 \rangle \quad \vec{n}_2 = \langle 2,-3,4 \rangle$

$\cos\theta = \frac{n_1 \cdot n_2}{|n_1|\,|n_2|} = \frac{2-3-4}{\sqrt{3}\,\sqrt{29}} \Rightarrow \theta \approx 122.4156$

or

$\theta \approx 57.5844°$

(25) $3x+y-4z=2$ (A) AND $3x+y-4z=24$ (B)

Choose $P_A = (0,2,0) \quad D = \frac{|3(0) + 1(2) - 4(0) - 24|}{\sqrt{3^2+1^2+4^2}} = \frac{22}{\sqrt{26}}$

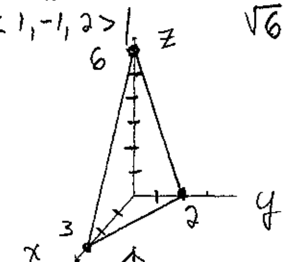
(26) $\ell: x = 1+t,\ y = 2-t,\ z = -1+2t$ and $P(0,0,0)$

Choose $Q: (1,2,-1)$ [t=0] $\quad \vec{QR} = \langle 1,-1,2 \rangle$ and $\vec{QP} = \langle 1,2,-1 \rangle$

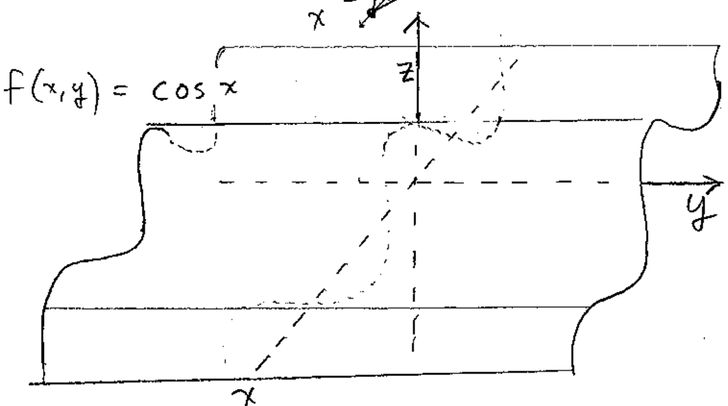
$R: (2,1,1)$ [t=1]

$d = \frac{|\vec{QR} \times \vec{QP}|}{|\vec{QR}|} = \frac{|\langle -3,3,-3 \rangle|}{|\langle 1,-1,2 \rangle|} = \frac{3\sqrt{3}}{\sqrt{6}} = \frac{3}{\sqrt{2}}$

(29) $f(x,y) = 6-2x-3y$



(30) $f(x,y) = \cos x$



(31) $f(x,y) = 4-x^2-4y^2$ Down paraboloid with z-MAX at $(0,0,4)$

(32) $f(x,y) = \sqrt{4-x^2-4y^2}$ Upper Half of an ELLIPSOID centered at $(0,0,0)$ with

$x^2+4y^2+z^2=4$

$r_x = 2 \quad r_y = 1 \quad r_z = 2$

(33) $y^2+z^2 = 1-4x^2$

$\frac{x^2}{1/4} + \frac{y^2}{1} + \frac{z^2}{1} = 1$

ELLIPSOID at $(0,0,0)$ $\quad r_x = \frac{1}{2} \quad r_y = 1 \quad r_z = 1$

(34) $y^2+z^2 = x$

PARABOLOID V(0,0,0)
+x is axis of symmetry

(35) $y^2+z^2=1$

CYLINDER with $r=1$ and x-axis as its axis

(36) $y^2+z^2 = 1+x^2$

$y^2+z^2-x^2=1$

HYPERBOLOID of 1 SHEET
x-axis as its axis

Chapter 9 Review (continued)

(37) $\boxed{(2, \pi/6, 2) \quad C \to S}$

$x = 2\cos(\pi/6) = \sqrt{3}$

$y = 2\sin(\pi/6) = 1$

$\left.\begin{array}{l} \rho^2 = (\sqrt{3})^2 + 1^2 + 2^2 = 8, \quad \rho = 2\sqrt{2} \\ \cos\phi = \frac{2}{2\sqrt{2}} \Rightarrow \phi = \pi/4 \end{array}\right\} (2\sqrt{2}, \pi/6, \pi/4)$

(38) $\boxed{(2, 2, -1)}$

$R \to C$

$\left.\begin{array}{l} r^2 = 2^2 + 2^2 = 8, \quad r = 2\sqrt{2} \\ \tan\theta = \frac{2}{2}, \quad \theta = \pi/4 \end{array}\right\} (2\sqrt{2}, \frac{\pi}{4}, -1)$

$R \to S$

$\left.\begin{array}{l} \rho^2 = 2^2 + 2^2 + (-1)^2 = 9, \quad \rho = 3 \\ \cos\phi = \frac{-1}{3} \Rightarrow \phi \approx 109.47° \text{ or } 1.9106 \end{array}\right\} (3, \pi/4, 1.9106)$

(39) $\boxed{(4, \pi/3, \pi/6)}$

$S \to R$

$x = 4\sin(\pi/6)\cos(\pi/3) = 1$

$y = 4\sin(\pi/6)\sin(\pi/3) = \sqrt{3}$

$z = 4\cos(\pi/6) = 2\sqrt{3}$

$S \to C$

$r^2 = 1^2 + (\sqrt{3})^2 = 4, \quad r = 2$

$\tan\theta = \frac{\sqrt{3}}{1}, \quad \theta = \pi/3$

R $(1, \sqrt{3}, 2\sqrt{3})$

C $(2, \pi/3, 2\sqrt{3})$

(40) a) $\theta = \pi/4$

C: Plane

S: ½-Plane

b) $\phi = \pi/4$

Cone: Vertex at origin +Z is its axis.

(41) $x^2 + y^2 + z^2 = 4$

C: $r^2 + z^2 = 4$

S: $\rho^2 = 4 \Rightarrow \rho = 2$

(42) $x^2 + y^2 = 4$

C: $r^2 = 4 \Rightarrow r = 2$

S: $x^2 + y^2 + z^2 = 4 + z^2$

$\rho^2 = 4 + \rho^2\cos^2\phi$

$\rho^2(1 - \cos^2\phi) = 4$

$\rho^2 \sin^2\phi = 4 \Rightarrow \boxed{\rho \sin\phi = 2 \text{ or } \rho = 2\csc\phi}$

(43) $z = 4y^2$

Rotated about z-axis yields the following

PARABOLOID: $z = 4x^2 + 4y^2$

$z = 4(x^2 + y^2)$

$\Rightarrow z = 4r^2$