MATH 1050 – Review For Final Exam

1.1 (51) $ax+b = 3(x-a)$

$ax+b = 3x-3a$

$3a+b = 3x-ax$

$3a+b = x(3-a)$

$\boxed{\frac{3a+b}{3-a} = x}$

1.2 (39) Tax $= .3(\$200{,}000)$

$= \$60{,}000$

Total Investment $= \$200{,}000 - \$60{,}000$

$= \$140{,}000$

Let x = \$ at 1.5%; y = \$ at 4%

$x+y = 140{,}000$

$.015x + .04y = 4350$

$\boxed{x = \$50{,}000,\ y = \$90{,}000}$

1.4 (17) $-4x^2 + x = -3$

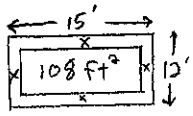
$-4x^2 + x + 3 = 0$

$4x^2 - x - 3 = 0$

$(4x+3)(x-1) = 0$

$\boxed{x = -3/4,\ x = 1}$

1.5 (21)



$A_{RUG} = L \cdot W = (15-2x)(12-2x) = 108$

$180 - 54x + 4x^2 = 108$

$4x^2 - 54x + 72 = 0$

$2x^2 - 27x + 36 = 0$

$(2x-3)(x-12) = 0$

$x = 3/2$ ~~$x = 12$~~

$L = 15-2(\frac{3}{2}) = 12$ Ft $\quad W = 12-2(\frac{3}{2}) = 9$ Ft

1.6 (27) $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

$\frac{1}{T} = \frac{1}{3} + \frac{1}{5} \quad (15T)$

$15 = 5T + 3T$

$15 = 8T$

$\boxed{T = \frac{15}{8} \text{ or } 1.875 \text{ Hrs Together}}$

(97) $\frac{E}{e} = \frac{R+r}{r}$ for e

$Er = eR + er$

$Er = e(R+r)$

$\boxed{\frac{Er}{R+r} = e}$

1.7 (37) At Least Break Even means

$R \geq C$

$105x \geq 85x + 900$

$20x \geq 900$

$\boxed{x \geq 45}$

1.8 (33) $4|x-3| > 12$

$|x-3| > 3$

$x-3 > 3$ OR $x-3 < -3$

$x > 6$ OR $x < 0$

$(-\infty, 0) \cup (6, \infty)$

(37) $|5-3x| \leq 7$

$5-3x \leq 7$ AND $5-3x \geq -7$

$-3x \leq 2$ AND $-3x \geq -12$

$x \geq -\frac{2}{3}$ AND $x \leq 4$

$\left[-\frac{2}{3}, 4\right]$

2.4 (65) (Year, Thousands #) $(0, 20)$ $(4, 4)$

Average Rate of Change $= m$

$= \frac{4-20}{4-0} = -4$

Value of Machine is DECREASING at \$4000/Yr

(76) $(1997, \$500)$ $(2007, \$90)$

A.R.C. $= \frac{90-500}{2007-1997} = -\$41$

The price of a new DVD has dropped an average of \$41/Yr from 1997 to 2007

2.5 (57) (% Women) $(1970, 43.3)$ $(2005, 59.3)$

$m = \frac{59.3-43.3}{2005-1970} = \frac{16}{35}$ or $.457$

$y = mx + b$

$43.3 = .457(1970) + b \Rightarrow b = -857$

$y = .457x - 857$

$y(2006) = \boxed{59.7\%}$

2.8 (38) $f(x) = 1-x^2$

a) $f(x+h) = 1-(x+h)^2$

$= 1-(x^2+2xh+h^2)$

$= 1-x^2-2xh-h^2$

b) $f(x+h) - f(x) = 1-x^2-2xh-h^2-(1-x^2)$

$= -2xh - h^2$

c) $\frac{f(x+h)-f(x)}{h} = \frac{-2xh-h^2}{h} = \frac{h(-2x-h)}{h} = \boxed{-2x-h}$

(49) $f \circ g(2)$

$= f[g(2)]$

$= f[3]$

$= 1$

(Table in Book is needed)

4.1 (67) $f(x) = \frac{x+1}{x-3}$ Find $f^{-1}(x)$

$y = \frac{x+1}{x-3}$

INVERSE: $x = \frac{y+1}{y-3}$

$x(y-3) = y+1$

$xy - 3x = y+1$

$xy - y = 3x+1$

$y(x-1) = 3x+1 \Rightarrow y = \frac{3x+1}{x-1}$

$\boxed{f^{-1}(x) = \frac{3x+1}{x-1}}$ Check: $F(4) = 5$

$F^{-1}(5) = \frac{16}{4} = 4$ ✓

4.2 (75) $A = P\left[1+\frac{r}{n}\right]^{nt}$

\$5000 $= P\left[1+\frac{.035}{4}\right]^{4\cdot 10}$

$5000 = P[1.00875]^{40}$

$\frac{5000}{1.00875^{40}} = P \approx \boxed{\$3528.81}$

(77) $1500 = 1200\left[1+\frac{r}{4}\right]^{4\cdot 5}$

$1.25 = \left[1+\frac{r}{4}\right]^{20}$

$\sqrt[20]{1.25} = 1+\frac{r}{4}$

$1.01122 - 1 = \frac{r}{4}$

$\boxed{4.4878\% \approx r}$

4.3 (67) $\log_2 \frac{ab}{cd}$

$= \boxed{\log_2 a + \log_2 b - \log_2 c - \log_2 d}$

(81) $\log_{10} 6 =$

$\log_{10}(2\cdot 3) = \log_{10} 2 + \log_{10} 3$

$= .3010 + .4771$

$= \boxed{.7781}$

4.3 (85) $\log_{10} \frac{9}{4}$

$= \log_{10} \frac{3^2}{2^2} = 2\log_{10} 3 - 2\log_{10} 2$

$= 2(.4771) - 2(.3010)$

$= .9542 - .6020$

$= \boxed{.3522}$

4.4 (61) $\log_2 5$

$= \frac{\ln 5}{\ln 2}$ OR $\frac{\log 5}{\log 2}$

$\approx \boxed{2.3219}$

4.5 (5) $3^x = 7$

$\ln 3^x = \ln 7$

$x \ln 3 = \ln 7$

$x = \frac{\ln 7}{\ln 3} \approx \boxed{1.771}$

(23) $3(2)^{x-2} + 1 = 100$

$3(2)^{x-2} = 99$

$2^{x-2} = 33$

$(x-2)\ln 2 = \ln 33$

$x - 2 = \frac{\ln 33}{\ln 2}$

$x = 2 + \frac{\ln 33}{\ln 2} \approx \boxed{7.044}$

(33) $\log(2-x) = .5$

$2-x = 10^{.5}$

$2 - 10^{.5} = x$

$\boxed{2-\sqrt{10} = x}$

4.6 (20) $A = Pe^{rt}$

$80{,}000 = 60{,}000e^{.0675t}$

$4/3 = e^{.0675t}$

$\ln(4/3) = .0675t$

$\frac{\ln(4/3)}{.0675} = t$

$t \approx 4.262$ years

5.1 (77) $y = ax^2 + bx + c$

$(2,3)$: $3 = 4a + 2b + c$ (X)

$(-1,0)$: $0 = a - b + c$ (Y)

$(-2,2)$: $2 = 4a - 2b + c$ (Z)

X−Z: $1 = 6b \Rightarrow b = 1/4$

X−Y: $3 = 3a + 3b$

$3 = 3a + 3(\frac{1}{4})$

$\frac{9}{4} = 3a \Rightarrow a = 3/4$

Y: $0 = \frac{3}{4} - \frac{1}{4} + c$

$c = -\frac{1}{2}$

$y = \frac{3}{4}x^2 + \frac{1}{4}x - \frac{1}{2}$

(88)

S	C
(320, 33)	$33 = 102400a + 320b + c$
(600, 40)	$40 = 360000a + 600b + c$
(1283, 50)	$50 = 1646089a + 1283b + c$

$C = aS^2 + bS + c$

$a = -.0000107567$ $b = .034896$ $c = 22.9347$

$C = -.0000107567S^2 + .034896S + 22.9347$

$45 = -.0000107567S^2 + .034896S + 22.9347$

$.0000107567S^2 - .034896 + 22.0653 = 0$

$S \approx 2383.48$ OR 860.63 knots

5.5 (13) $y = x^2 + 4x$

$2x - y = -8$

$2x - (x^2 + 4x) = -8$

$2x - x^2 - 4x = -8$

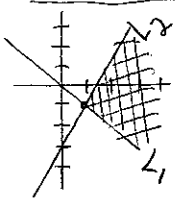
$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4$, $y = 0$ $\quad$ $x = 2$, $y = 12$

5.6 (29) $x + y \geq 0 \rightarrow y \geq -x$ L_1

$2x - y \geq 3 \rightarrow y \leq 2x - 3$ L_2



7.1 (1) $a_n = 4n + 10$

$a_1 = 4(1) + 10 = 14$

$a_2 = 4(2) + 10 = 18$

$a_3 = 4(3) + 10 = 22$

$a_4 = 4(4) + 10 = 26$

$a_5 = 4(5) + 10 = 30$

(26) $a_1 = 2$, $a_2 = 5$, $a_n = a_{n-1} + a_{n-2}$

$a_3 = a_2 + a_1 = 5 + 2 = 7$

$a_4 = a_3 + a_2 = 7 + 5 = 12$

7.2 (45) Find the sum of the 1st 50 Positive Odds

$1 + 3 + 5 + 7 + 9 + \cdots a_{50}$

$a_n = a_1 + (n-1)d$

$a_{50} = 1 + (50-1)2 = 99$

$S_n = \frac{n}{2}[a_1 + a_n]$

$S_{50} = \frac{50}{2}[1 + 99] = 2500$

(72) $30 + 29 + 28 + 27 + \cdots$ (a_{30})

$S_n = \frac{n}{2}[a_1 + a_n]$

$S_{30} = \frac{30}{2}[30 + 1]$

$S_{30} = 465$

7.3 (11) $-4, -12, -36, -108, \cdots$

$a_n = a_1 r^{n-1} \Rightarrow a_n = -4(3)^{n-1}$

$a_5 = -4(3)^{5-1} = -324$

(17) $a_2 = -6$ $a_7 = -192$

$\frac{a_7}{a_2} = \frac{-192}{-6} = r^5$

$\Rightarrow r = 2$

$a_1 = \frac{a_2}{r} = \frac{-6}{2} = -3$

(64) Distance Falling Downward $= S_1 = 10 + 7.5 + 5.625 + 4.21875 + \cdots$ ($r = .75$)

$S_\infty = \frac{a_1}{1-r} = \frac{10}{1-3/4} = 40$

Distance Bouncing Upward $= S_2 = 7.5 + 5.625 + 4.21875 + \cdots$ ($r = .75$)

$S_\infty = \frac{7.5}{1-3/4} = 30$

Total Distance Travelled $= S_1 + S_2 = 40 + 30 = 70$ m

7.4 (42) Eighth term of $(2c - 3d)^{14}$

$\binom{14}{7}(2c)^7(-3d)^7 = 3432(128c^7)(-2187d^7)$

$= -960740352c^7d^7$

7.6 (35) 15 in club, How many ways to choose: President, VicePresident, Secretary?

Without Replacement, Order Matters $n = 15$, $r = 3$

${}_{15}P_3 = \frac{15!}{12!} = 2730$

(39) 40 members of Banker's Association. How many different groups of 6 are possible?

Without Replacement, Order does not matter $n = 40$, $r = 6$

${}_{40}C_6 = \frac{40!}{6!34!} = 3{,}838{,}380$