MATH 1050 - Chapter 4 REVIEW

4.1

(67a) $f(x) = \boxed{y = \frac{x+1}{x-3}}$

INVERSE $x = \frac{y+1}{y-3}$

$x(y-3) = y+1$

$xy - 3x = y+1$

$xy - y = 3x+1$

$y(x-1) = 3x+1$

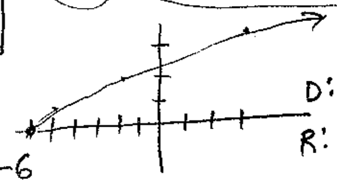
$y = \frac{3x+1}{x-1}$

$\boxed{f^{-1}(x) = \frac{3x+1}{x-1}}$

$f(4) = \frac{4+1}{4-3} = 5$

$f^{-1}(5) = \frac{15+1}{5-1} = 4$ ✓

(69c) $\boxed{f(x) = y = \sqrt{6+x} \quad x \geq -6}$



	$f(x)$	$f^{-1}(x)$
D:	$[-6, \infty)$	$[0, \infty)$
R:	$[0, \infty)$	$[-6, \infty)$

(81) $\boxed{\text{Find } f^{-1}(-3)}$

Find $f(?) = -3$

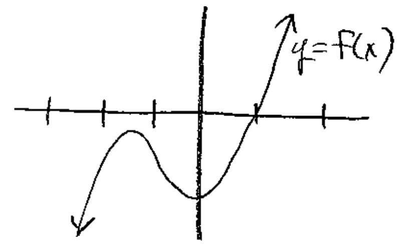
↑

-2

Because $f(-2) = -3$

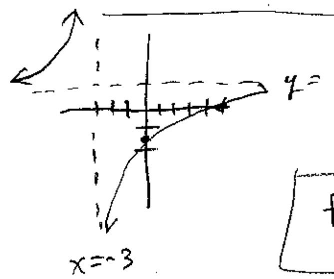
we know $f^{-1}(-3) = -2$

(87) $\boxed{f(x) = 6x^3 + 11x^2 - 6}$



Fails HORIZONTAL-LINE TEST, Therefore this function is NOT 1-to-1

(89) $\boxed{f(x) = y = \frac{x-5}{x+3}}$ Yes, 1-to-1 (Passes HORIZ line Test)



INV! $x = \frac{y-5}{y+3}$

$y(x-1) = -3x-5$

$\boxed{f^{-1}(x) = \frac{3x+5}{1-x}}$ Follow steps in #67

Check: $f(-2) = -7$; $f^{-1}(-7) = -2$ ✓

4.2 $\boxed{g(x) = \left(\frac{1}{4}\right)^x}$

(10) $g\left(\frac{3}{2}\right) = \left(\frac{1}{4}\right)^{3/2} = \left(\sqrt{1/4}\right)^3 = \left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{8}}$

(57) $\boxed{27^{4z} = 9^{z+1}}$

$(3^3)^{4z} = (3^2)^{z+1}$

$\cancel{3}^{12z} = \cancel{3}^{2z+2}$

$12z = 2z+2$

$10z = 2$

$\boxed{z = 1/5}$

(75) $A = P\left[1 + \frac{r}{n}\right]^{nt}$

$5000 = P\left[1 + \frac{.035}{4}\right]^{\cancel{4(10)}\ 40}$

$5000 = P(1.4169)$

$\boxed{\$3528.81 = P}$

(93) $f(x) = a^x \quad \overset{x}{(-3}, \overset{y}{64)}$

$y = a^x$

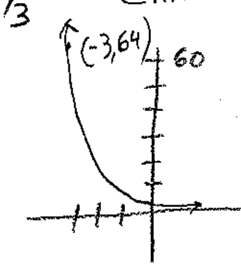
$64 = a^{-3}$

$(64)^{-1/3} = (a^{-3})^{-1/3}$

$\frac{1}{4} = a$

$\boxed{f(x) = \left(\frac{1}{4}\right)^x}$

CHECK



4.3 (7) $\boxed{\log_6 36 = 2} \Longrightarrow 6^2 = 36$

(13) $x = \log_5 \frac{1}{625}$

$x = \log_5(5^{-4}) \Rightarrow \boxed{x = -4}$

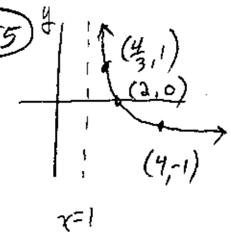
(27) $\boxed{x = \log_4 16^{1/3}}$

$x = \log_4 (4^2)^{1/3}$

$x = \log_4 (4^{2/3})$

$\boxed{x = 2/3}$

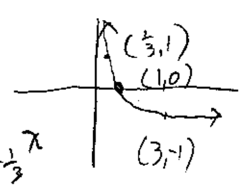
(55)



$f(x) = A\log_b(Bx+C)+D$

$C = -1$

$\boxed{f(x) = \log_{\frac{1}{3}}(x-1)}$



$y = \log_{\frac{1}{3}} x$

(65) $\boxed{\begin{aligned} \log(AB) &= \log A + \log B \\ \log\left(\frac{C}{D}\right) &= \log C - \log D \\ \log(x^n) &= n \log x \end{aligned}}$

3 Basic Log Properties

$\log_m \left(\frac{5r^3}{z^5}\right)^{1/2} = \log_m \frac{5^{1/2} r^{3/2}}{z^{5/2}} = \frac{1}{2}\log_m 5 + \frac{3}{2}\log_m r - \frac{5}{2}\log_m z$

4.3 (75) $\boxed{2\log_m a - 3\log_m b^2} = \boxed{\log_m \frac{a^2}{b^6}}$

4.4 Change of Base Theorem

$\log_b x = \frac{\ln x}{\ln b}$ OR $\frac{\log x}{\log b}$

4.5 (13) $6^{x+1} = 4^{2x-1}$

$\ln 6^{x+1} = \ln 4^{2x-1}$

$(x+1)\ln 6 = (2x-1)\ln 4$

$x\ln 6 + \ln 6 = 2x\ln 4 - \ln 4$

$\ln 6 + \ln 4 = 2x\ln 4 - x\ln 6$

$\ln 6 + \ln 4 = x[2\ln 4 - \ln 6]$

$\frac{\ln 6 + \ln 4}{2\ln 4 - \ln 6} = x \approx \boxed{3.24017}$

(73) $A = P\left[1 + \frac{r}{n}\right]^{nt}$

$30{,}000 = 27{,}000\left[1 + \frac{.04}{4}\right]^{4t}$

$1.11\overline{1} = [1.01]^{4t}$

$\ln(1.11\overline{1}) = 4t\ln(1.01)$

$\frac{\ln(1.11\overline{1})}{4\ln(1.01)} = t$

$\boxed{t \approx 2.647 \text{ years}}$

4.6 (7) $y = y_0 e^{kt}$

$A(t) = e^{-.00043t}$

FIND $\frac{1}{2}$ LIFE: $\frac{1}{2} = e^{-.00043t}$

$\ln\left(\frac{1}{2}\right) = -.00043t$

$\frac{\ln(1/2)}{-.00043} = t$

$\boxed{\tfrac{1}{2}\text{-LIFE: } t = 1611.97 \text{ years}}$

(63) $\boxed{\log_8(.59)} = \frac{\ln(.59)}{\ln(8)} \approx \boxed{-.2537}$

(45) $\ln(4x-2) - \ln 4 = -\ln(x-2)$ D: $x > 2$

$\ln(4x-2) + \ln(x-2) = \ln 4$

$\ln[(4x-2)(x-2)] = \ln 4$

$\cancel{\ln}[4x^2 - 10x + 4] = \cancel{\ln} 4$

$4x^2 - 10x + 4 = 4$

$4x^2 - 10x = 0$

$2x(2x-5) = 0$

$\cancel{x = 0}$ $\boxed{x = \frac{5}{2}}$

Not in Domain

(13) For Carbon-14 $y = y_0 e^{-.0001216t}$

15% Remains: $.15 = 1e^{-.0001216t}$

$\ln(.15) = -.0001216t$

$\frac{\ln(.15)}{-.0001216} = t$

$\boxed{t \approx 15601.3 \text{ years}}$